Name ____ Test 1, Spring 2021

1) Find the product below. (15 points)

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & 7 & 1 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & -2 \\ 0 & 2 & 5 \\ -1 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1-2 & 4 & -2+6 \\ 3-1 & 12+14 & -6+35+3 \\ -2-1 & -8 & 4+3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & 4 \\ 2 & 26 & 32 \\ -3 & -8 & 7 \end{bmatrix}$$

2) Row reduce the matrix below to reduced echelon form. (15 points)

$$\begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 8 & 11 \\ 4 & 10 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 8 & 11 \\ 4 & 10 & 14 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 3 & 8 & 11 \\ 4 & 10 & 14 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 3 & 8 & 11 \\ 4 & 10 & 14 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 2 & 2 \\ 4 & 10 & 14 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

$R_1 \to \frac{1}{2}R_2$ $\quad R_2 \to R_2 - R_1$ $\quad R_3 \to R_3 - 3R_1$ $\quad R_4 \to R_4 - 4R_1$

$$\sim_R \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_4$ $\quad R_2 \to \frac{1}{2}R_2$ $\quad R_3 \to R_3 - 2R_2 R_1 \to R_1 - 2R_2$

3) Find the null space of the matrix below. (15 points)

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & -4 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -4 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

$x_4 \in \mathbb{R}$
$x_3 = 4x_4$
$x_2 = -2x_4$
$x_1 = x_4$

$$span\left(\left\{\begin{bmatrix} 1 \\ -2 \\ 4 \\ 1 \end{bmatrix}\right\}\right)$$

4) Express the span below in set builder notation. Do not include redundant vectors. (10 points)

$$span\left(\left\{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -4 \end{bmatrix}\right\}\right)$$

$$\left\{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x_3 : x_1, x_2, x_3 \in \mathbb{R}\right\}$$

5) Answer the following questions. (3 points each)

A) Let A be a 4×4 matrix which, when row reduced, has 4 pivots. How many solutions can the equation $A\vec{x} = \vec{0}$ have?

1

B) Let A be a 4×4 matrix which, when row reduced, has 3 pivots. How many solutions can the equation $A\vec{x} = \overrightarrow{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}}$ have?

0 or ∞

C) Let $A\vec{x} = \vec{0}$ be a system of 5 equations in 3 variables. If the row space of A is $\mathbb{R}^3$, how many solutions can the system have?

1

D) Let A be a 6×7 matrix for which $A\vec{x} = \vec{b}$ with $\vec{b} \neq \vec{0}$ has no solutions. When row reduced, what is the maximum number of pivots A can have?

5

E) Let A be a 3×3 matrix that is a product of elementary matrices. Does A have an inverse?

Yes

6) Multiply the matrices below. (6 points)

$$\left[\begin{array}{c|cccc}0&1&0&0&0\\2&0&1&0&0\\0&0&0&1&0\\4&0&0&0&1\\ \hline 5&0&0&0&0\end{array}\right]\left[\begin{array}{ccccc}2&0&1&0&0\\ \hline 2&3&4&5&3\\6&7&8&9&3\\1&2&3&4&4\\5&6&7&8&5\end{array}\right]$$

$$\begin{bmatrix}\begin{bmatrix}0&0&0&0&0\\4&0&2&0&0\\0&0&0&0&0\\8&0&4&0&0\end{bmatrix}+\begin{bmatrix}2&3&4&5&3\\6&7&8&9&3\\1&2&3&4&4\\5&6&7&8&5\end{bmatrix}\\ \begin{bmatrix}10&0&5&0&0\end{bmatrix}+\begin{bmatrix}0&0&0&0&0\end{bmatrix}\end{bmatrix}=\begin{bmatrix}\begin{bmatrix}2&3&4&5&3\\10&7&10&9&3\\1&2&3&4&4\\13&6&11&8&5\end{bmatrix}\\ \begin{bmatrix}10&0&5&0&0\end{bmatrix}\end{bmatrix}=\begin{bmatrix}2&3&4&5&3\\10&7&10&9&3\\1&2&3&4&4\\13&6&11&8&5\\10&0&5&0&0\end{bmatrix}$$

By partitioning the left matrix, most of the arithmetic is avoided.

7) Multiply the matrices below (6 points)

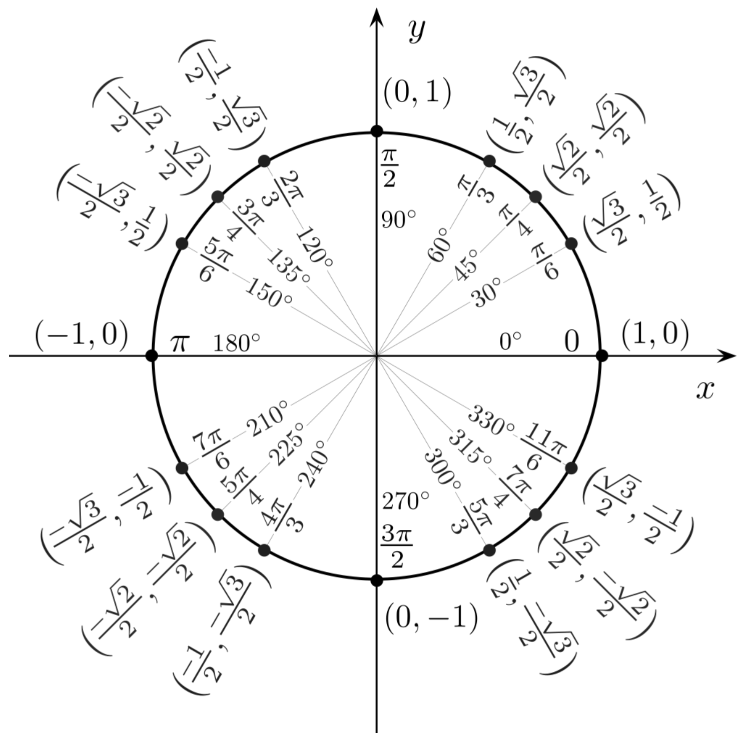
$$\begin{bmatrix}1&0&0&0&0\\1&1&0&0&0\\0&0&1&0&0\\0&0&0&2&0\\0&0&0&0&0\end{bmatrix}\begin{bmatrix}2&0&1&0&0\\2&3&4&5&3\\6&7&8&9&3\\1&2&3&4&4\\5&6&7&8&5\end{bmatrix}$$

$$\begin{bmatrix}2&0&1&0&0\\4&3&5&5&3\\6&7&8&9&3\\2&4&6&8&8\\0&0&0&0&0\end{bmatrix}$$

By using the fact that the left matrix is the product of three* elementary matrices, most of the arithmetic is avoided.

8) Find the angle between the two vectors below. You may use the unit circle provided here. (6 points)

$$\vec{v} = \begin{bmatrix}1\\2\\2\\1\end{bmatrix}, \vec{w} = \begin{bmatrix}-1\\3\\0\\0\end{bmatrix}$$



$$\left\|\begin{bmatrix}1\\2\\2\\1\end{bmatrix}\right\| \left\|\begin{bmatrix}-1\\3\\0\\0\end{bmatrix}\right\| \cos(\theta) = \begin{bmatrix}1\\2\\2\\1\end{bmatrix} \bullet \begin{bmatrix}-1\\3\\0\\0\end{bmatrix}$$

$\sqrt{1+4+4+1}\sqrt{1+9}\cos(\theta) = -1+6$

$\sqrt{10}\sqrt{10}\cos(\theta) = 5$

$10\cos(\theta) = 5$

$\cos(\theta) = \frac{1}{2}$

$\theta = \frac{\pi}{3} = 60^0$

9) Solve the system of equations below. (6 points)

$$\begin{aligned} x_1 \quad - \ x_3 &= 5 \\ x_2 + 2x_3 &= 4 \end{aligned}$$

$x_3 \in \mathbb{R}$
$x_1 = 5 + x_3$
$x_2 = 4 - 2x_3$

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} x_3 + \begin{bmatrix} 5 \\ 4 \\ 0 \end{bmatrix} : x_3 \in \mathbb{R} \right\}$$

10) How many solutions does matrix equation below have? (6 points)

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

∞